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Fred E.J. Linton* (FLinton@Wesleyan.edu), Math. Dept., 649 Science Tower, Wesleyan Univ., Middletown, CT 06459. *A Hahn-Banach Theorem Converse*. Preliminary report.

A mild refinement of the Hahn-Banach Theorem helps characterize the real Banach spaces from amongst the abstract algebras whose operations are the natural operations, finitary and infinitary, on Banach discs. These algebras, sometimes called convexoids, constitute the varietal reflection of the category of Banach spaces; the operations are all the “sub-convex-combination” operators arising from absolutely summable real sequences (finite or infinite) with $(l^{(1)})$ norm ≤ 1 .

The Hahn-Banach Theorem, commonly read as asserting that the natural “evaluation map” $i_V : V \rightarrow V^{**}$ from any real Banach space V to its second dual V^{**} is an isometric embedding, is easily tweaked to reveal that i_V is actually an equalizer (or “difference kernel”) of the corresponding evaluation map $i_{V^{**}} : V^{**} \rightarrow V^{****}$ for V^{**} and the second transpose $(i_V)^{**} : V^{**} \rightarrow V^{****}$ of i_V itself.

Counterparts of these maps persist for convexoids V , and our converse is then:

*The convexoid V is (the unit disc of) a real Banach space if (and only if) the map i_V is an equalizer of the pair $(i_{V^{**}}, (i_V)^{**})$.* (Received August 03, 2006)